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LETTER

Polarization Dependence of Pure Bending Loss in Slab Optical Waveguides

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SUMMARY The finite-difference beam-propagation method is applied to the analysis of a bent step-index slab optical waveguide. The results obtained in the rectangular coordinates with a modified index profile are compared with those in the cylindrical coordinates with a real index profile. It is found that the attenuation constant for TM₀ mode is larger than that for TE₀ mode. The polarization dependence of bending loss is negligible, provided the refractive index difference is less than 2%.

key words: beam-propagation method, bent waveguides, polarization

1. Introduction

Considerable effort has been made to evaluate bending loss of dielectric slab waveguides [1]–[14]. In addition to the WKB method and perturbation method, the beam-propagation method [5]–[7], [9]–[14] is currently used to analyze bent optical waveguides. However, most of works treated TE modes, and the bending loss for TM modes received little attention [13], [15].

The purpose of this letter is to evaluate the pure bending loss (attenuation constant) of a bent step-index slab waveguide, and to clarify the polarization dependence. In the analysis, we use the finite-difference beam-propagation-method (FD-BPM) based on the Crank-Nicholson scheme [16]–[18]. The BPM analysis has the advantage that it gives information on the transient properties of fields, when a bent waveguide is excited with a guided mode of a straight waveguide.

After confirming the validity of use of an equivalent straight waveguide with a modified index profile, we assess the accuracy of the transverse differential operator with index change derived by Stern [19]. Numerical simulation for TM mode in a bent waveguide shows that the deformed electric field propagates having discontinuity at the interface between the core and cladding. It is found that the attenuation constant for TM₀ mode is larger than that for TE₀ mode.

2. Formulation

A step-index slab waveguide with a core width of $2D$ is considered. The refractive indices of the core and cladding are designated as N_{CO} and N_{CL} , respectively.

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N_{CL} is fixed to be unity and the normalized frequency is chosen to be $V = kD(N_{CO}^2 - N_{CL}^2)^{1/2} = 1.5$, where k is the wavenumber in free space. The wavelength $\lambda = 1.55\mu\text{m}$ is used throughout this analysis. The bending radius is designated as R . We use the normalized bending radius [2]–[4] defined as

$$R_n = 2N_{CO}k(1 - N_{CL}^2/N_{CO}^2)^{3/2}R. \quad (1)$$

We have two approaches for investigating the propagating field and the attenuation constant in a bent optical waveguide. One is to use the cylindrical coordinate system with a real index profile [13], [14]. The other is to use the rectangular coordinate system together with a modified index profile for an equivalent straight waveguide [20]. We first compare the two approaches in TE mode.

In the cylindrical coordinates shown in Fig. 1 (a), we get the Fresnel equation expressed as

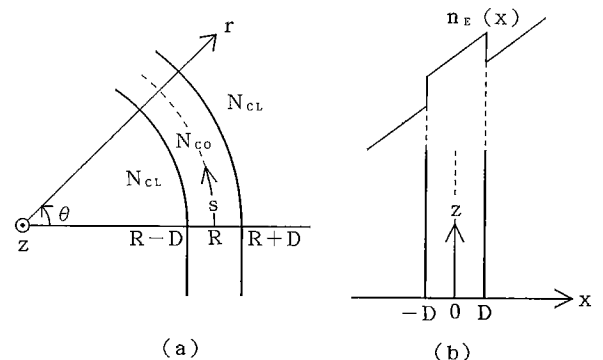
$$2jkn_0 \frac{R^2}{r^2} \frac{\partial \phi}{\partial s} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + k^2 \left[n^2 - n_0^2 \frac{R^2}{r^2} \right] \phi. \quad (2)$$

where n_0 is the reference index, which is taken to be that in the cladding.

In the rectangular coordinates shown in Fig. 1 (b), we get the Fresnel equation expressed as

$$2jkn_0 \frac{\partial \phi}{\partial z} = \frac{\partial^2 \phi}{\partial x^2} + k^2 [n^2 - n_0^2] \phi. \quad (3)$$

The index profile of the waveguide is transformed to



(a) cylindrical coordinates with a real index profile.
(b) rectangular coordinates with a modified index profile.

Fig. 1 Configuration and coordinate system of a bent step-index waveguide.

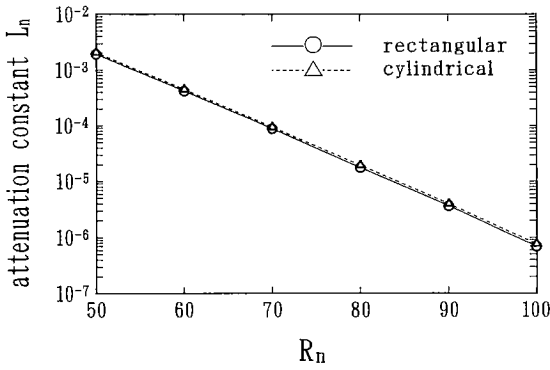


Fig. 2 Comparison between the normalized attenuation constants obtained in the cylindrical coordinates with a real index profile and in the rectangular coordinates with a modified index profile.

that of an equivalent straight waveguide. If the bending radius R is much larger than the core width, the index profile of the equivalent straight waveguide can be expressed as

$$n_E(x) = n(x)[1 + (x/R)], \quad (4)$$

where $n(x)$ is the real index profile of the waveguide and x is perpendicular to the core axis.

Maruta and Matsuhara [11] analyzed Eq. (2) using the Galerkin method. In this paper, we solve each of Eqs. (2) and (3) by the FD-BPM to easily compare both results. Application of the Crank-Nicholson scheme to Eq. (2) is straightforward [14]. The transparent boundary condition [21] can be imposed at the edge of the computational window. As the initial field, the fundamental-mode field of the straight waveguide is used.

For convenience, we use the following normalized attenuation constant L_n per unit radian [2]–[4],

$$L_n = \alpha R(1 - N_{CL}^2/N_{CO}^2)^{1/2}. \quad (5)$$

Figure 2 shows a comparison between the normalized attenuation constants calculated by Eqs. (2) and (3). The parameters are $N_{CO}/N_{CL} = 1.03$, $\Delta x = \Delta r = D/20 = 0.07497 \mu\text{m}$, $\Delta z = \Delta s = 0.5 \mu\text{m}$, and the total number of sampling points in the transverse direction is $M_T = 1024$. It is worth mentioning that good correlation is found to exist between both results even in relatively small bending radii. In this paper, we adopt the rectangular coordinates with the modified index profile for the following investigation.

Consideration is next given to the TM mode analysis. Care has to be taken when we treat TM modes, since the transverse differential operators include the index change. The formulation can be made by either the transverse electric or magnetic field. For the E formulation based on the transverse electric field, we have to evaluate

$$\frac{\partial}{\partial x} \left(\frac{1}{n^2} \frac{\partial}{\partial x} (n^2 E_x) \right).$$

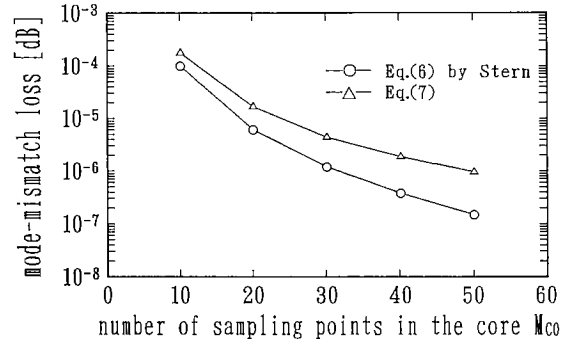


Fig. 3 Comparison in the mode-mismatch loss of a straight step-index waveguide.

For the H formulation based on the transverse magnetic field, we have to evaluate

$$n^2 \frac{\partial}{\partial x} \left(\frac{1}{n^2} \frac{\partial H_y}{\partial x} \right).$$

To evaluate the transverse differential operators, Huang et al. [17] and Liu et al. [18] employed the expression derived by Stern [19]. For the H formulation, we have

$$n^2 \frac{\partial}{\partial x} \left(\frac{1}{n^2} \frac{\partial H_y}{\partial x} \right) = \frac{a_i H_{i-1} - 2b_i H_i + c_i H_{i+1}}{\Delta x^2}, \quad (6)$$

where a_i , b_i and c_i are defined in Ref. [19]. Similar operator can also be obtained for the E formulation. Alternatively, we can directly derive the following finite-difference expression instead of Eq. (6):

$$n^2 \frac{\partial}{\partial x} \left(\frac{1}{n^2} \frac{\partial H_y}{\partial x} \right) = \frac{1}{\Delta x^2} ((N_i + 1)H_{i+1} - 2H_i - (N_i - 1)H_{i-1}), \quad (7)$$

where

$$N_i = \frac{n_i^2}{4} \left(\frac{1}{n_{i+1}^2} - \frac{1}{n_{i-1}^2} \right).$$

To assess the accuracy of Eqs. (6) and (7), we evaluate the mode-mismatch loss in a straight waveguide. The mismatch between input field and calculated output field is known to be a sensitive indicator of the accuracy of a beam-propagation method [16]. Figure 3 shows the comparison in the mode-mismatch loss as a function of the number of sampling points in the core, $M_{CO} (= 2D/\Delta x)$. As M_{CO} is increased, Δx decreases and M_T increases, since the computational window dimension is fixed to be $76.772 \mu\text{m}$. Other numerical parameters are the same as those in Fig. 2. As expected, the accuracy improves as M_{CO} is increased. It is interesting to note that the result obtained from Stern's expression has better accuracy than that from Eq. (7). Hence, we adopt the Stern's expression in the following analysis.

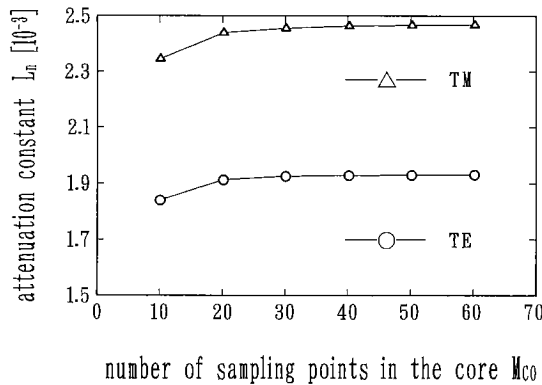


Fig. 4 Normalized attenuation constant as a function of the number of sampling points in the core.

3. Results

Before discussing the polarization dependence, we first investigate the convergence of numerical results, since the accuracy of the numerical results depends on the size of the discretization mesh. Figure 4 shows the normalized attenuation constant L_n as a function of M_{CO} . The normalized bending radius R_n is taken to be 50, and the computational window dimension is fixed to be $76.772\mu\text{m}$, which gives $M_T = 1024$ for $M_{CO} = 40$. Other geometrical and numerical parameters are the same as those in Fig. 2. It is seen that both results for the TE and TM modes tend to converge as M_{CO} is increased. It can be said that the sufficient value of M_{CO} is 40, which corresponds to $\Delta x = D/20 = 0.07497\mu\text{m}$. Further calculation shows that the results are not sensitive to the change in Δz , as long as $\Delta z \leq 0.5\mu\text{m}$. We adopt $\Delta x = D/20$, $\Delta z = 0.5\mu\text{m}$, and $M_T = 1024$ in the following analysis.

A typical example of the propagating electric field for TM mode is shown in Fig. 5. The bent waveguide with $R_n = 50$ is excited with the field of the fundamental mode TM_0 of the straight waveguide, which can be obtained analytically. It should be noted that the electric field is discontinuous at the interface between the core and cladding.

Since the mode of a straight waveguide differs from that in a bent waveguide, the strong radiation occurs when the incident field enters the bent waveguide. This radiation is closely related to the transition loss. As the field propagates, the field tends to exhibit a steady state, in which the field deforms and its maximum shifts towards the outer side of the bend. The field deformation results in the pure bending loss (attenuation constant).

Figure 6 shows the normalized attenuation constant L_n as a function of the normalized bending radius R_n . As expected, L_n becomes large as R_n is decreased. It is found that L_n for TM_0 mode is larger than that for TE_0 mode.

Figure 7 shows L_n as a function of N_{CO}/N_{CL} .

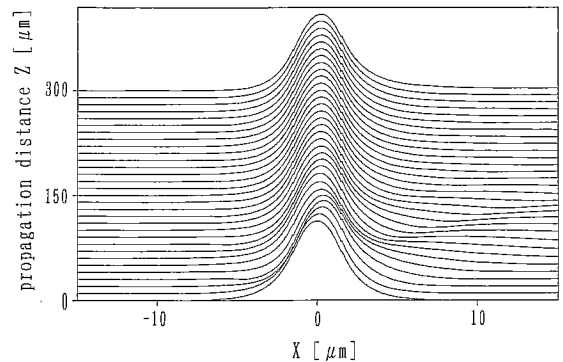


Fig. 5 Propagating electric field when the bent waveguide is excited with the field of TM_0 mode in the straight waveguide.

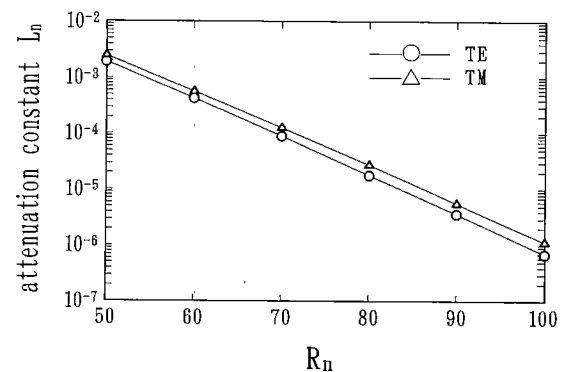


Fig. 6 Normalized attenuation constant as a function of normalized bending radius R_n .

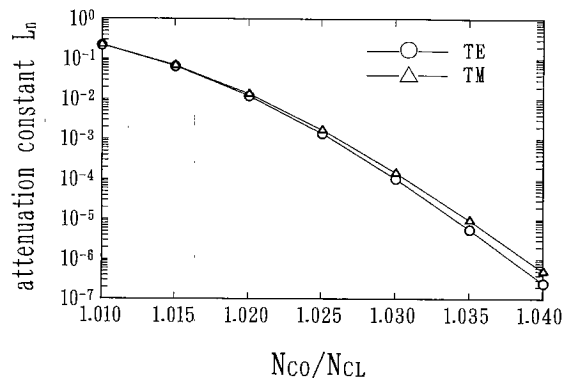


Fig. 7 Normalized attenuation constant as a function of refractive index of N_{CO}/N_{CL} .

In this calculation, the bending radius is $R = 600\mu\text{m}$, which corresponds to $R_n = 68.9$ for $N_{CO}/N_{CL} = 1.03$. As N_{CO}/N_{CL} is increased, L_n decreases. It is obvious that the polarization dependence of bending loss becomes large as N_{CO}/N_{CL} is increased. The difference in bending loss for the TE and TM modes is negligible, provided the refractive index difference is less than 2%. In other words, the scalar analysis is enough to evaluate the bending loss if the index difference is less than 2%.

4. Conclusions

The propagating field and the pure bending loss in a slab optical waveguide have been investigated by the finite-difference beam-propagation method. It is numerically confirmed that the attenuation constant evaluated in the rectangular coordinates with a modified index profile for an equivalent straight waveguide agrees well with that in the cylindrical coordinates with a real index profile. Preliminary calculation also shows that the transverse differential operator with the index change can be accurately calculated by the Stern's expression. It is found that the attenuation constant for TM_0 mode is larger than that for TE_0 mode. The polarization dependence of bending loss is negligible, provided the refractive index difference is less than 2%.

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